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INTERACTIVE MULTIPLE GOAL PROGRAMMING: METHOD AND APPLICATION.

By Peter Nijkamp* and Jaap Spronk**

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1. INTRODUCTION

During the last decade multidimensional optimization models have received a lot of attention from mathematicians, economists and planners. The heterogeneity of policy issues, the existence of goal conflicts, the multilevel (hierarchical) organization of the decision apparatus, the incommensurable character of a wide variety of objectives, and the lack of information on trade-offs between diverging priorities have evoked the need of modern decision tools such as multi-objective programming models and multi-criteria analyses. Several of these methods follow more or less the approach via achievement levels which was earlier developed in goal programming analyses.

Recently, however, these goal programming models have been further extended toward interactive learning models. In the present paper, a new variant of a learning decision-model, labeled Interactive Multiple Goal Programming (I.M.G.P.) will be presented. Next, several aspects of interactive learning models will be discussed. Special attention will be paid to a set of criteria which may characterize each model from the class of interactive decision models. The applicability of interactive multiple goal programming will be illustrated by means of a numerical example in the field of portfolio management. Finally a number of actual and potential applications of interactive multiple goal programming techniques will be presented, followed by a brief evaluation and outlook of further research.

2. INTERACTIVE MULTIPLE GOAL PROGRAMMING

2.1. Introduction.

Interactive Multiple Goal Programming (I.M.G.P.) is a further extension of the well-known goal programming approach initiated and further developed by Charnes and Cooper. These authors present an overview of the development of the field of goal programming in a recent study (see Charnes and Cooper [1977]). Our approach is mainly taken from a survey report by Nijkamp and Spronk [1977].

In general, a multiple goal program can be formulated as:

$$(2.1.)\quad \begin{cases} \text{Minimize } f(\underline{y}^+ , \underline{y}^-) \\ \text{subject to} \\ \underline{g}(\underline{x}) - \underline{y}^+ + \underline{y}^- = \underline{b} & , \\ \underline{x}\varepsilon R, R = \{\underline{x} \mid \underline{h}(\underline{x}) \leqslant \underline{h}\} & , \\ \underline{y}^+ , \underline{y}^- \geqslant \underline{0} & , \\ \text{and } y_i^+ \cdot y_i^- = 0 \quad \text{for } i=1,\dots,m \end{cases}$$

where f is the (dispreference) function with the positive (y_i^+) and the negative (y_i^-) deviations from the aspired levels (b_i) of the goal variables $g_i(\underline{x})$ as arguments ($i=1,\dots,m$). The feasible area R of the instrumental variables $\underline{x}$ is bounded by the set of constraints $\underline{h}(\underline{x})$. In general, the minimand f is assumed to be convex. The feasible area is also assumed to be convex. In many cases, both the goal variables $\underline{g}(\underline{x})$ and the constraints $\underline{h}(\underline{x})$ are assumed to be linear in $\underline{x}$. We then have:

$$(2.2.)\quad \begin{cases} \underline{g}(\underline{x}) = A.\underline{x} \\ \underline{h}(\underline{x}) = B.\underline{x} \end{cases}$$

where A is a matrix of order (m x n), B is a matrix of order (k x n), and $\underline{x}$ is a n-dimensional vector. As shown in Nijkamp and Spronk [1977, pp.7-9], the following general form for the function f can be deduced from the Minkovski metric:

$$(2.3.)\qquad f(y^+, y^-) = \left\{ \sum_{i=1}^{m} \alpha_i^+ . \left(\left(\frac{y_i^+}{b_i}\right)\right)^p + \sum_{i=1}^{m} \alpha_i^- . \left(\left(\frac{y_i^-}{b_i}\right)\right)^p \right\}^{1/p}$$

It is easily seen, that (2.3.) is a weighted (by α_i^+ and α_i^-) and standardized form of the l_p metric. That is, for p=1 we get an absolute value metric, for p=2 an Euclidean metric, and for $p \to \infty$ an approximation of the Chebychev (minimax) metric. In multiple goal programming, the weighing factors α_i^+ and α_i^- may be replaced by preemptive priority factors, by which lexicographic orderings can be handled (Ibid, p.16).

In our opinion, goal programming is one of the stronger multi-objective programming models available. It has a close correspondence with decision-making in practice. Furthermore, it has some attractive technical properties.

Several empirical findings from decision-making practice are, in our opinion, rather convincing to demonstrate the practical usefulness of multiple goal programming. As mentioned by several authors, the method corresponds fairly well to the results of the behavioral theory of the firm. In practice, decision-makers are aiming at various goals, formulated as aspiration levels. The intensity with which the goals are strived for may vary for each goal; in other words, different 'weights' may be assigned to different goals[1]. The use of aspiration levels in decision-making is also reported by scientists from other fields, for instance, psychology (see for a short overview Monarchi et al [1976]). In the same way, also preemptive priorities are known in real life problems. Support for this essentially lexicographic viewpoint is provided by Fishburn [1974] and Monarchi et al [1976]. A more concrete example of the correspondence of multiple goal programming and practice is provided by Ijiri [1965], who regards multiple goal programming as an extension of break-even analysis, which is widely used in business practice.

The above plea for multiple goal programming is of a somewhat theoretical nature. However, the operational usefulness of multiple goal programming has also been recognized in practice, as shown by the many applications which have been reported in literature (see Nijkamp and Spronk [1977] for an overview).

One of the technical advantages of multiple goal programming is that there is always a solution for a well-defined problem, even if some goals are conflicting, provided the feasible region R is non-empty. This is due to the inclusion of the deviational variables y_i^+ and y_i^- . These variables show whether the goals are attained or not, and in the latter case they measure the distance between the realized and aspired goal levels. Another advantage of multiple goal programming is that it does not require very sophisticated solution procedures. Especially the linear goal programming problems can be solved by easily available linear programming routines.

An important drawback of multiple goal programming is its need for fairly detailed a priori information on the decision-maker's preferences.

1) As shown by Lane ([1970], pp.57-60), the correspondence of the behavioral theory and multiple goal programming is not complete, because the latter gives a specific interpretation of 'satisfying goals as close as possible'.

Goal programming requires the definition of aspiration levels, the partition into preemptive priority classes and the assessment of weights within these classes. We agree with those scholars advocating interactive approaches. These methods are based on a mutual and successive interplay between a decision-maker and an analyst. Interactive methods neither require an explicit representation or specification of the decision-maker's preference function nor an explicit quantitative representation of trade-offs among conflicting objectives. Obviously, the solution of a decision problem requires that the decision-maker provides information about his priorities regarding alternative feasible states, but in normal interactive procedures only limited information is asked for which more over can be provided in a stepwise manner. The task of the analyst is to display all relevant information especially about admissable values of the criteria and reasonable compromise solutions. Unfortunately, most of the usual interactive approaches lack some of the advantages of 'traditional' multiple goal programming, such as for instance the possibility to include preemptive priorities. Furthermore multiple goal programming can handle situations of satisficing behaviour in contrast with most existing interactive methods. This situation, combined with the often shown power of the traditional approach to include piecewise linear functions (cf. Charnes & Cooper [1977], justifies the effort to seek for an interactive variant of the traditional approach.

2.2. A Brief Description of Interactive Multiple Goal Programming.

In this subsection we present the general lines of a new, interactive variant of multiple goal programming (I.M.G.P.). In the next subsection we list some of the main features of I.M.G.P. A more detailed description is given in an earlier report (Nijkamp and Spronk [1978b]).

I.M.G.P. is capable of including all advantages of multiple goal programming. For instance, preemptive priorities and piecewise linear functions can be handled in a straightforward way. Furthermore, the interactive process imitates practice in formulating aspiration levels, assessing priorities, seeking for a solution and readjustment of the aspiration levels. The method needs no more a priori information on the decision-maker's preference structure than other interactive multi-objective programming models. However, all available a priori information can be incorporated within the procedure.

Step 0 - First identify the goal variables $g_i(\underline{x})$, $i=1,\ldots,m$ as linear or piecewise linear functions of $\underline{x}$, the vector of instrumental variables $x_1,x_2,\ldots,x_n$. We assume the $g_i(\underline{x})$ to be concave in $\underline{x}$. Then specify the feasible set R, which is assumed to be convex and within which an optimal solution must be found. When the decision-maker's preferences could be described by a preference function f (note however, that we do not make any attempt in this direction), this function should be a concave function of both $g_i(\underline{x})$, $i=1,\ldots,m$ and x_i, $i=1,\ldots,n$. An optimal solution is then defined by:

$$(2.4.)\begin{cases}\text{Max } f = f\{g_i(\underline{x}), i=1,\ldots,m\}, \text{ subject to}\\ \underline{x} \in R.\end{cases}$$

To simplify this brief exposition, we assume further

$$(2.5.)\quad \frac{\partial f}{\partial g_i} > 0 \text{ for } i=1,\ldots,m,$$

so that we presuppose a higher value of each of the goal variables is preferred to a lower value of (the same) goal variable[2].

Step 1 - Next maximize successively each of the m goal variables $g_i(\underline{x})$ separately and denote the maxima by g_i^* and the m corresponding combinations of the instrumental variables by $\underline{x}_i^*$, $i=1,\ldots,m$. It is not possible to find a feasible value of $g_i(\underline{x})$ which exceeds g_i^*. On the other hand, it is not necessary to accept a value of $g_i(\underline{x})$ which is lower than g_i^{min}, defined as:

$$(2.6.)\quad g_i^{min} = \text{Min}_{j=1}^{m} \{g_i(\underline{x}_j^*)\},$$

the lowest value of $g_i(\underline{x})$ resulting from the successive maximizations of the goal variables. In I.M.G.P. we define a 'solution' $\underline{S}$ as a vector of minimum values imposed on each of the goal variables. Therefore, it is clear that a final solution $\underline{S}^*$ must be found between the 'ideal' (but mostly infeasible) solution $\underline{I}$, and the 'pessimistic' solution $\underline{Q}$, which are defined respectively as:

2) In the full description of I.M.G.P. it is shown that cases in which $\partial f/\partial g_i$ is negative and cases in which f is not a monotone function of the $g_i(\underline{x})$ can also be included.

$$(2.7)\quad \begin{cases} \underline{I} = [\, g_1^*, g_2^*, \ldots, g_m^* \,] & \text{and} \\ \underline{Q} = [\, g_1^{\min}, g_2^{\min}, \ldots, g_m^{\min} \,] \end{cases}$$

To facilitatethe notation we have included the optimistic solution $\underline{I}$ and the pessimistic solution $\underline{Q}$ in the (2 x m) 'potence matrix' P.

Step 2 - For each goal variable $g_i(\underline{x})$, the decision-maker may have defined aspiration levels $g_{ij}, j=2,\ldots,k_i-1$; with the following property

$$(2.8)\quad g_i^{\min} < g_{i2} < g_{i3} < \ldots < g_{ik_i-1} < g_i^*$$

Furthermore we define

$$(2.9)\quad \begin{cases} g_{i1} = g_i^{\min} & \text{and} \\ g_{ik_i} = g_i^* \end{cases}$$

In the following steps these goal values are used in constructing trial solutions $\hat{S}_i$ which have to be evaluated by the decision-maker. Because proposed goal levels are sometimes considered as being too high, we need the auxiliary vector $\underline{\delta}$, whose elements $\delta_j, j=1,\ldots,m$ correspond to the m goal variables. We define δ_j as the difference of the lowest level of $g_j(\underline{x})$ being rejected by the decision-maker and the highest level of $g_j(\underline{x})$ being accepted thus far. At the first stage of the procedure, no proposals have been made and consequently, no goal level has been rejected. Therefore we put $\delta_j = 0$ for $j=1,\ldots,m$ during the first step.

Step 3 - Define the starting solution as:

$$(2.10)\quad \underline{S}_1 = [\, g_{11}, g_{21}, \ldots, g_{m1} \,],$$

which is thus equal to the pessimistic solution defined in (2.7) Present this solution together with the potence matrix P_1 to the decision-maker.

Step 4 - If the proposed solution is satisfactory for the decision-maker, one may accept it; if not, continue with step 5. Define R_i as the subset of R defined by the goal levels in $\underline{S}_i$.

Step 5 - The decision-maker then has to answer the following question: "Given the provisional solution $\underline{S}_i$, which goal variable should be improved first?"[3)]

Step 6 - Let us assume that the decision-maker wants to augment the j'th goal variable. Then construct a new trial solution $\hat{\underline{S}}_{i+1}$, which differs with respect to $\underline{S}_i$ only as far as the value of the j'th goal variable is concerned (denoted by $g_j(\underline{x})_{\hat{\underline{S}}_{i+1}}$ and $g_j(\underline{x})_{\underline{S}_i}$ respectively).

<u>If</u> $\delta_j = 0$ no proposed value of $w_j(\underline{x})$ has been rejected thus far, by which we can propose the next higher aspiration level listed in step 2. <u>If</u> $\delta_j > 0$, a value of $g_j(\underline{x})$ which exceeds the current solution by an amount δ_j has been rejected by the decision-maker. In this case, define:[4)]

(2.11) $$g_j(\underline{x})_{\hat{\underline{S}}_{i+1}} = g_j(\underline{x})_{\underline{S}_i} + \tfrac{1}{2}.\delta_j$$

When a provisional value for $g_j(\underline{x})$ has been calculated in one of both above mentioned ways, we introduce the restriction:

(2.12) $$g_j(\underline{x}) \geqslant g_j(\underline{x})_{\hat{\underline{S}}_{i+1}}$$

and proceed to step 7.

Step 7 - Join the restriction formulated in step 6 or in step 9 to the set of restrictions describing the feasible region R_i. Next calculate a new potence matrix, like in step 2, but subject to the new set of restrictions. Label this potence matrix $\hat{P}_{i+1}$.

Step 8 - Confront the decision-maker with $\underline{S}_i$ and $\hat{\underline{S}}_{i+1}$ on one hand and with P_i and $\hat{P}_{i+1}$ on the other hand. The shifts in the potence matrix can be viewed as a 'sacrifice' for reaching the proposed solution. <u>If</u> the decision-maker judges this sacrifice to be justified, accept the proposed solution by putting $\underline{S}_{i+1} = \hat{\underline{S}}_{i+1}$ and $P_{i+1} = \hat{P}_{i+1}$.

3) After step 9 we discuss the case in which the decision-maker wants to raise more than one goal variable at the same time.
4) At this moment , the decision-maker may wish to define a new aspiration level. In our opinion, it is wise to give him explicitely the opportunity to do so.

Furthermore, in the computer algorithm (see figure 2.1.), put $\delta_j = \frac{1}{2}.\delta_j$. (which is only relevant for $\delta_j > 0$). If the decision-maker considers the sacrifice unjustified, the proposed value of $g_j(\underline{x})$ is obviously too high. Therefore, drop the constraint added in step 7 and proceed to step 9.

Step 9 - We now know that $g_j(\underline{x})_{\underline{S}_i}$ is too low and that $g_j(\underline{x})_{\hat{\underline{S}}_{i+1}}$ is too high in the decision-maker's view. By definition, we thus may set δ_j equal to the difference between these two values. A new proposal value $\hat{\underline{S}}_{i+1}$ is then calculated[5)] by defining:

$$g_j(\underline{x})_{\hat{\underline{S}}_{i+1}} = g_j(\underline{x})_{\underline{S}_i} + \tfrac{1}{2}.\delta_j. \tag{2.13}$$

Like in step 6 we add the restriction that $g_j(\underline{x})$ must equal or exceed the new proposal value and go to step 7 in order to calculate a new potence matrix $\hat{P}_{i+1}$.

When the decision-maker is not able to indicate which single goal variable should be augmented, we assume he is at least capable of defining a set of goal variables whose values need to be augmented. In this case, the procedure must be modified slightly. This is shown in figure 2.1. where we give a flow chart of the procedure.

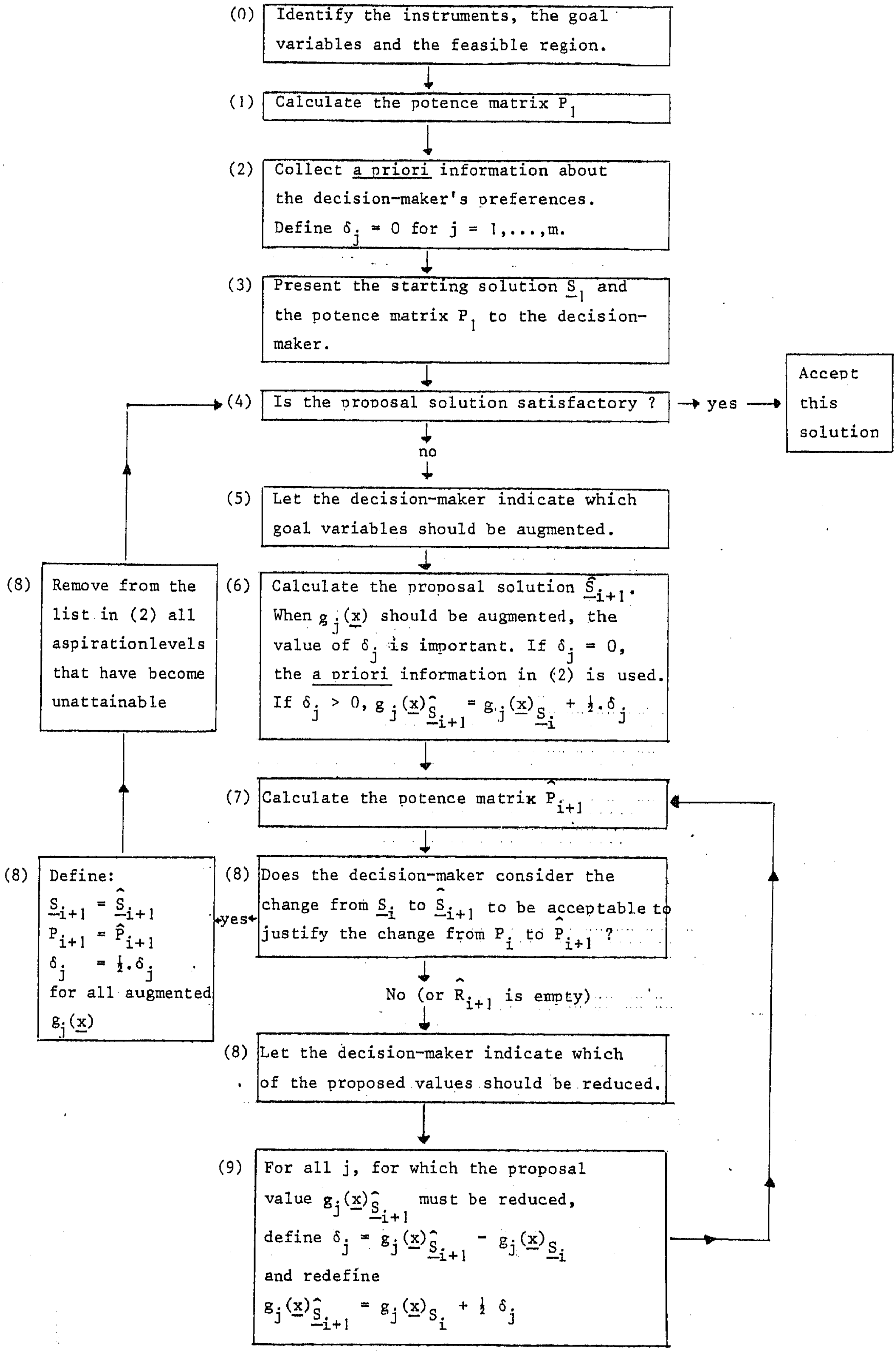
2.3. Main Features and Possibilities of the Procedure

In this subsection some key properties and possibilities of I.M.G.P. will be mentioned.

* In I.M.G.P. the goal variables are assumed to be known and concave in the instrumental variables. The preference function of the decision-maker is not assumed to be known. However, it is assumed to be concave, both in the goal variables and in the instrumental variables. Clearly, these assumptions are not very restrictive. For instance, both optimizing and satisficing behaviour can be incorporated.

5) Also in this case the decision-maker himself may wish to define a new aspiration level.

Figure 2.1. A flow chart of the extended interactive goal programming procedure



* The decision-maker only has to give information on his local preferences. This is done on basis of a solution and a potence matrix presented to him. A solution is a vector of minimum values for the respective goal variables. The potence matrix shows for each of these goal variables separately the maximum value, given the solution concerned. The decision-maker only has to indicate whether a solution is satisfactory or not, and if not, which of the minimum goal values should be raised. He does not have to specify how much these goal values should be raised. Nor is there any need to specify weighing factors.[6)] A new solution is presented to him together with a new potence matrix. He then has to indicate whether the shifts in the solution outweigh the shifts in the potence matrix. If not, a new solution is calculated and so on.

* I.M.G.P. needs no more a priori information than other interactive programming models. However, all available a priori information can be incorporated within the procedure. Especially aspiration levels and preemptive priorities which have been defined by the decision-maker can be incorporated in the interactive process quite easily (note that I.M.G.P. offers the decision-maker the opportunity to reconsider this a priori information during the interactive process). This ability to include a priori information makes I.M.G.P. also suitable for multicriteria problems which are repetitive and not important enough to warrant a permanent intervention of the decision-maker.

* As shown in Nijkamp and Spronk [1978b], I.M.G.P. converges within a finite number of interations to a final solution, which exists and is feasible. Apart from an ε-neighbourhood, this solution is optimal. Whether this solution is unique or not, depends on the decision-maker's preferences. (For instance, when the decision-maker is a satisficer having formulated targets which are attainable within the feasible region, a unique final solution does not exist in general).

* The decision-maker is assumed to be able to answer the questions posed by I.M.G.P. His answers must be consistent, although he is allowed to make some

6) On the other hand, if the decision-maker is able to provide such information it can be included in a straightforward manner. Moreover, in I.M.G.P. he gets an opportunity to reconsider such an aspiration level vis ã vis the shifts in potence caused by imposing it.

errors during the interactive process. Finally, because of the possible learning effects, the procedure must be repeated several times to be sure that a final solution is found which is as close as possible to the optimum.

* Given a new (trial) solution , the maxima of the goal variables must be (re)-calculated during each interation of I.M.G.P. This can be done with the help of any optimization method which meets the fairly unrestrictive requirements imposed by I.M.G.P. (i.e. convexity[7]) of the feasible region R and concavity of the preference function and the goal variables). Nevertheless, it may be advantageous to formulate the problem in linear terms. Then, I.M.G.P. can make a straightforward use of goal programming routines. In that case for each proposal solution, a set of goal programs can be formulated, which differs mutually only with respect to one element in the objective function (see Nijkamp and Spronk [1978b], subsection 4.1). When a new solution is proposed, these goal programs are only modified with respect to some of the right-hand side constants, being the re-specified goal levels altered. The linear format of I.M.G.P. has all advantages of standard multiple goal programming as discussed in subsection 2.1. Especially the easy available dual information should be noticed (see Isermann [1977] and Nijkamp and Spronk [1977]).

3. SOME IMPORTANT ELEMENTS OF AN INTERACTIVE DECISION MODEL

Many normative models in economics, management science and other academic fields depart from the well-known assumption that the decision-maker's preferences can be specified, at least in principle, by means of a global preference function. In practical situations, the specification of these preferences often appears to be a very intricate and sometimes even impossible task. In such cases, interactive decision models may offer a way out because they only require partial local information on the decision-maker's preferences. However, interactive decision models are sometimes criticized for several reasons. Contrary to the conventional methods, interactive methods are said to encompass the danger to manipulate the decision-maker toward a final solution which is not optimal with respect to his global preferences, especially because

7) However, with some minor modifications I.M.G.P. can also be applied to discrete decision models (see Nijkamp and Spronk [1978a].

these interactive methods may be regarded as a learning process in which the decision-maker reacts upon the questions raised to him. From this viewpoint, the specific method of obtaining information from a decision-maker may influence his answers. Although this criticism completely side-steps the advantages of a learning process[8], the point raised should certainly be born in mind.

Others argue that not all information is used in an efficient manner, because convergence of a particular interactive method might be speeded up by adding a few behavioural assumptions. The latter suggestion is, however, not a criticism upon the method as such, but adds a useful complementary proposal.

These few points show that the evaluation of an interactive decision model itself is a multi-criteria problem. In order to illustrate this statement, we shall propose now some criteria which may be important in evaluating interactive decision models. In doing so, we will adopt some of the criteria proposed in studies of Cohon and Marks [1975], Roy [1971, 1976 ab], Wallenius [1975] and Wallenius and Zionts [1976].

The criteria to be used in the evaluation of an interactive method can be divided into three main classes, which are related to:

I. The kind of problems for which the method is applicable

II. The communication process between decision-maker and decision-model

III. Technical properties of the interactive process.

These criteria will now be described in more detail.

I. <u>The kind of problems for which the method is applicable</u>.

The judgement of whether and when to apply a given interactive method can be facilitated by means of a reference to a suitable problem classification. Starting from an analogous framework for a normative theory of decision-aid suggested by Roy [1976 ab], we propose to include the following features in such a classification.

a. <u>The set of alternative actions, A</u>:

- this set may be either limited or flexible in size over time.
- the elements of this set are mutually exclusive or not.

8) A major drawback of the conventional methods is that they assume the existence of an omniscient decision-maker, who thus has nothing left to learn from the existing situation.

b. The set of impacts of a particular action a:
   - the number of elementary impacts.
   - the scale chosen to represent the dimension associated with an elementary outcome (quantitative, qualitative, e.g.).

c. The objective to select from A:
   - one and only one action considered as 'best'.
   - all actions corresponding to efficient solutions.
   - all actions corresponding to satisficing solutions.

d. The nature of the criteria related to the dimensions
   - criteria may be pseudo-, quasi-, pre- or true criteria (see Roy [1976a], p. 14).
   - the degree of measurability of the criteria.

e. The nature of global preferences
   - the *a priori* assumptions concerning the decision-maker's preference structure (for example, can satisficing behaviour be included or not and are efficient solutions required or not?).

f. The possibility to include incomplete and/or fuzzy information

II. The communication process between decision-maker and decision-model

Clearly, the way in which the process of interaction between decision-maker and model (possibly guided by an analyst) has been structured, is of crucial importance for the acceptance of the interactive method by the decision-maker. For instance, when only limited information is provided to the decision-maker, he may feel that he is being kept ignorant. Likewise, he will require some freedom to influence the direction of the interactive process.

To characterize the communication process, we first suggest to use the following qualifications:

a. The questions posed to the decision-maker.
   - Is he asked to answer 'zero-one' questions (e.g.,'Do you prefer this solution to the preceeding one?') or does he have to specify cardinal information (e.g., the specification of a marginal rate of substitution between two goal variables)?
   - Is he asked to answer these questions with respect to weights or with respect to achievement levels attached to the goal variables?

- What is the number of questions to be answered at each iteration?
  To stress the importance of including these qualifications, we quote Cohon and Marks [1975,p. 209]: "It is futile, and antithetical to the essence of planning, to complicate the analysis with all sort of esoteric terms and terminology. Yet, as a subsequent discussion will show, some of the multiobjective techniques rely on the collection of abstruse, sometimes exotic data from the decision-maker, thereby producing meaningless results".

b. <u>The information for the decision-maker.</u>
   - Is the decision-maker confronted with a fixed amount of information or does he have some options to select from the available information those items which he believes to be interesting?
   - Does the model provide dual (or similar other) information, given the state of the problem?
   - Is he given one or more solutions at a time?

c. <u>Options available to the decision-maker to control the interactive process</u>
   - What are the decision-maker's degrees of freedom to change the direction of the search process for a consecutive solution?
   - In case the decision-maker changes his mind during the interactive procedure, can he turn it back to an earlier solution?

d. <u>Perception of the method by the decision-maker.</u>

   The evaluation of an interactive method presupposes also some idea of how the method is perceived by the decision-maker.
   Wallenius [1975] carried out a laboratory experiment in order to compare from a human decision-maker's point of view the performance of three interactive methods. From this experiment we adopt the following measures, which obviously are of a more subjective nature then the preceeding ones.
   - The decision-maker's confidence in the best compromise.
   - The ease of using the method.
   - The ease of understanding the logic of the method.
   - The use of the information provided to aid the decision-maker.
   - The speed of convergence (in our opinion the decision-maker is sensible for the number of evaluations he has to make before a final solution is reached due to both the time spent of each evaluation and the computertime (= waiting time) between each solution).

III. Technical properties of the interactive process.

a. The specific solution (optimization) procedure(s) which has to be used in the computational phases of the process.

b. Computertime per iteration.
This is not only important for the acceptance of the method by the decision-maker (see II.d), but also as a cost factor.
Of course, this measure depends on the problem to be solved, the solution (optimization) procedure chosen and last but not least the type and size of the computer.

c. Convergence properties.
- Is the procedure convergent or not?
- If the procedure is convergent, is there a unique final solution or not?
- What are the minimum and maximum number of iterations needed to approach such a final solution close enough? (This depends on the accuracy chosen, but also on the number of goal variables, the preference structure and the properties of the set of possible actions A).

We conclude this section by typifying the method described in subsection 2.2 (I.M.G.P.) in terms of the criteria mentioned above. First, we outline the kind of problems which can be handled by interactive multiple goal programming. The set of feasible actions (R) is given and fixed over time. However, in the case this set changes over time, the interactive procedure has not to be started all over again. This is because then the solution obtained for the unchanged (old) problem can be used to make an advanced start. The set R needs to be convex. However, with the loss of some attractive properties of I.M.G.P., also mutually exclusive actions can be handled (see section 5). In I.M.G.P., the goal variables (criteria) are assumed to be measurable and known functions of the instrumental variables (actions). The already mentioned example in section 5 also shows how to include a criterion which has to be measured on an ordinal scale. The assumptions on the global preferences of the decision-maker are very weak. The preference function of the decision-maker (which has not be explicated) is assumed to be concave, both in the goal variables and in the instrumental variables. This implies, that I.M.G.P. can be used (depending on the desires of the decision-maker), to generate a unique final solution, an efficient solution or a satisficing solution. The method is not suitable to generate the complete set of efficient solutions.

Instead, it is aimed at finding the efficient solution, which is considered (by the decision-maker) as being the 'best' element within the set of all efficient solutions.

In I.M.G.P., the communication process between decision-maker and decision-model has been structured in a way, which has some attractive properties. The decision-maker only has to provide a limited amount of information, although he has the option to give more information whenever he wants to do so. At each iteration, the model providesa large amount of information concerning the state of the problem. The decision-maker may choose which information has to be displayed and which not. Finally, the decision-maker has some options to command the interactive process. For instance, by formulating aspiration levels during the process. At this stage, we can not say very much about the perception of the method by the decision-maker in practice. Although some contacts have been made (see section five) and the first reactions are certainly not discouraging, no final reports can be given on this moment. However, in the next section we illustrate the applicability of I.M.G.P. by means of a numerical example in the field of portfolio management. In this example, we pay special attention to the method's convergence speed.

About the technical properties of I.M.G.P. we can be very short. In the computational phases of I.M.G.P. any solution procedure which meets the not very restrictive requirements mentioned above (convexity of the feasible region R and concavity of the goal variables and the global preference function). As already mentioned in subsection 2.3, I.M.G.P. converges within a finite number of iterations to a final solution, which is known to exist and to be feasible. The computertime per iteration and the number of iterations needed to reach a final solution depends, among other things, on the problem to be solved and the solution procedure chosen. Again, the example in the next section may give a first impression.

4. I.M.G.P. APPLIED TO PORTFOLIO CONSTRUCTION: A NUMERICAL EXAMPLE

Quite a few authors have formulated the problem of portfolio construction as a multiple goal program. Baum and Carlson [1974] showed how efficient solutions can be found. However, they did not indicate how a final solution must be chosen from the set of efficient solutions. Lee [1972], Lee and Lerro [1973], Kumar, Philippatos and Ezzell [1978] and Bronsema and van de Kieft [1978] formulated the problem as a standard multiple goal problem, requiring complete a priori information on the decision-maker's preferences. In this section we show an example, which has been adopted from Lee [1972], to illustrate that, by means of I.M.G.P., a unique (optimal and efficient) solution can be found without any a priori information on the decision-maker's preferences. Furthermore, this illustration is used to discuss the convergence properties of I.M.G.P. Here, we do not enter into the merits of the way in which the portfolio problem has been formulated. For these, we refer to the authors mentioned above.

4.1. Lee's Model.

Lee ([1972], Ch.9) formulated a model for selecting efficient mutual funds portfolios. Given a budget constraint, a set of legal constraints and a set of restrictions representing the maximum allowable concentration in any industry, two goals must be strived for. The first goal is to maximize the expected return of the portfolio, E_p. The second goal is to minimize the risk of the portfolio, as measured by the variance in the returns of the portfolio, V_p. Because the calculation of V_p requires the collection of an enormous amount of information (especially the covariances between the returns of the securities) and to avoid the computational burden of a quadratic goal program, the linear approximation formula provided by Sharpe [1967] was used. According to Sharpe, the uncertain return of a security is linearly related (apart from a disturbance term) to an index representing the general tendency of the stock market.
In this apprach, the risk of a portfolio consisting of n securities is represented by:

$$(4.1) \qquad B_p = \sum_{i=1}^{n} x_i . B_i ,$$

where x_i is the part of the budget invested in security i, B_i is the market volatility of security i and B_p the market volatility of the portfolio. The market volatility of a security or a portfolio measures it's relationship with some stock market index common to all securities. The market volatility (or beta) of a security is often called the systematic risk of a security. Besides this risk each security has it's own unsystematic risk component, viewed as a random variable with a mean of zero and standard deviation of s_i. All pairs of random variables representing the unsystematic risk are assumed to have zero covariance and to be uncorrelated with the market index. (Thus, the covariance between securities is only explained by their relationship to the market index). Sharpe has demonstrated that, given a sufficient degree of diversification (i.e. a number of securities exceeding 20), the unsystematic risk factors can be neglected, by which the representation in (4.1) becomes a good approximation of the total risk of the portfolio. Nevertheless, Lee wanted to account for the unsystematic risk of the portfolio. Therefore, he introduced 'current dividend yield' as an additional goal variable 'to help us compensate for having the income fund suffer some unaccounted for risk' (ibid. pp. 227-228).

Given this framework, Lee formulated both a goal program for a growth portfolio and a goal program for an income portfolio, only different with respect to their objective functions. The first objective function was formulated to maximimize expected growth, to minimize market volatility and to minimize current dividend yield in this order which was moreover preemptive. The second objective function was also ordered by preemptive priority factors, but in the reverse direction.

In the I.M.G.P. approach we replaced these two objective functions by three objective functions, one for each goal variable. Furthermore, we dropped some variables and constraints which were apparently redundant. For each of the goal variables we introduced a new constraint to indicate the minimum values required by the decision-maker. Finally, the example we used could be formulated as follows:

(4.2a) $\text{Min } (y_1^-)$ (Maximize expected growth)

(4.2b) $\text{Min } (y_2^+)$ (Minimize market volatility)

(4.2c) $\text{Min } (y_3^-)$ (Maximize current dividend)

s.t. (4.3a) $y_1^- \leqslant S_1$

(4.3b) $y_2^+ \leqslant S_2$

(4.3c) $y_3^- \leqslant S_3$

(To indicate the minimum goal values required by the decision-maker, we used the S_i to indicate the maximally allowed distance from the target goal levels)[9]

and (4.4) $\sum_{i=1}^{53} E_i . x_i + y_1^- = 2$ (Target of expected growth)

9) Each time the I.M.G.P. procedure was started these S_i values were given a large value to be sure that these constraints were non-binding at the start. During the procedure, these S_i values were repeatedly changed by 'the decision-maker'.

$$(4.5) \quad \sum_{i=1}^{53} B_i \cdot x_i - y_2^+ = 0 \qquad \text{(Target of market volatility)}$$

$$(4.6) \quad \sum_{i=1}^{53} D_i \cdot x_i + y_3^- = 1 \qquad \text{(Target of current dividend)}$$

$$(4.7) \quad \sum_{i=1}^{53} x_i \leqslant 1 \qquad \text{(budget constraint)}$$

$$(4.8) \quad x_i \leqslant 0.05 \qquad \text{(up to 5\% legal constraints)}$$

$$(4.9) \quad \sum_{i=5}^{12} x_i \leqslant 0.25 \qquad \text{(diversification constraint, Electronics \& Instrument Industry)}$$

$$(4.10) \quad \sum_{i=13}^{20} x_i \leqslant 0.25 \qquad \text{(Cosmetics \& Drugs)}$$

$$(4.11) \quad \sum_{i=21}^{26} x_i \leqslant 0.25 \qquad \text{(Air Transport)}$$

$$(4.12) \quad \sum_{i=38}^{43} x_i \leqslant 0.25 \qquad \text{(Oils)}$$

$$(4.13) \quad \sum_{i=47}^{53} x_i \leqslant 0.25 \qquad \text{(Utilities)}$$

As input for the models we used the data given by Lee. These data are listed in appendix A of this report. For further details on these data we refer to Lee (ibid. pp. 232-235).

4.2. A Solution by Means of I.M.G.P. and an Imaginary Decision-Maker.

In order to demonstrate some of the convergence properties of I.M.G.P., we introduce an 'imaginary decision-maker'. We assume that his preference structure can be described adaquately by a preference function which is known to us but not to him. Nevertheless, we assume him to be able to give his judgments concerning a solution proposed to him by the I.M.G.P. procedure, and moreover to do so in a manner which is in complete accordance with the preference function specified.

Given the a priori knowledge of the decision-maker's preference function, we can calculate an optimal solution of the portfolio problem described in the preceeding section. At the same time, we can execute the I.M.G.P. procedure by 'interacting' with the imaginary decision-maker. Then, the solution obtained from the a priori specified preference function can be compared with the solution resulting from the interactive procedure.

Below, we present the results for the case in which the decision-maker's preference function can be specified by:

$$(4.14) \quad f(E_p, B_p, D_p,) = 0.4 \times E_p + 0.2 \times B_p + 0.4 \times D_p,$$

where E_p, B_p and D_p are the portfolio's expected growth, market volatility and current dividend yield respectively. For the present, we have assumed that the decision-maker did not specify any aspiration level. This assumption will be discussed after the presentation of the results. Before, one more remark should be made. In the I.M.G.P. procedure, the decision-maker has to indicate, given an accepted solution, which goal value should be raised. In the present case, we introduced the simplifying assumption, that the decision-maker first tries to drive the first goal variable to it's optimal value, then the same with the second and the third goal variable respectivily. Because of the linear nature of the preference function (4.14), this assumption causes no serious difficulties. However, in the case a non-linear preference function is used instead of (4.14), this simplifying assumption must be dropped. We stopped to change the value of a given goal variable as soon as it reached a value which was sufficiently close[10] to the optimal value.

The computer program for the described experiment has been run on an I.B.M. 370/158 computer of the Technical University in Delft, the Netherlands. In the program, which has been written in PL/I, the MPSX package has been used to solve the linear programming problems.[11] As can be gathered from the problem formulation in the preceeding subsection, the successive solutions were originally stated in terms of the deviational variables y_1^-, y_2^+ and y_3^-.

10) We considered a given goal value close enough to it's optimal value as soon as the deviation from the optimal value became equal or less than the distance between the optimal value and the target level specified in (4.4)-(4.6), divided by 400.

11) We are indebted to Drs J. Hartog for his almost unlimited willingness to answer our questions on PL/I and on the MPSX package. Next, we would like to thank Mr. C. Ouwerkerk, for his kind assistance in running and adapting the computer programs.

Table 4.1. I.M.G.P. Applied to Lee's Model.

	description of the objective function	expected growth		market volatility		current dividend		comment
		*	**	*	**	*	**	
(a)	solution for (4.14)	0.4975		0.5844		0.00285		
(b)	growth portfolio (Lee)	0.677		1.361		0.009		
(c)	income portfolio (Lee)	0.105		0.922		0.033		
(d)	max.expected growth	0.677		1.361		0.009		
(e)	min.market volatility	0.0		0.0		0.0		
(f)	max.current dividend	0.107		0.887		0.033		
(0)	initial pessimistic sol.	0.0		1.361		0.0		
(1)	proposed constraints	0.338		1.361		0.0		$\delta_1=0.338$
	pessimist.solutions		0.338		1.361		0.0027	
	potential solutions		0.677		0.346		0.029	accepted
(2)	proposed constraints	0.507		1.361		0.0027		$\delta_1=0.169$
	pessimistic solutions		0.507		1.361		0.0029	
	potential solutions		0.677		0.608		0.025	rejected
(3)	proposed constraints	0.423		1.361		0.0027		$\delta_1=0.085$
	pessimistic solutions		0.507		1.361		0.0023	
	potential solutions		0.677		0.462		0.028	accepted
(4)	proposed constraints	0.465		1.361		0.0023		$\delta_1=0.043$
	pessimistic solutions		0.465		1.361		0.0026	
	potential solutions		0.677		0.471		0.026	accepted
(5)	proposed constraints	0.486		1.361		0.0026		$\delta_1=0.021$
	pessimistic solutions		0.486		1.361		0.0028	
	potential solutions		0.677		0.565		0.0268	accepted
(6)	proposed constraints	0.497		1.361		0.0028		$\delta_1=0.011$
	pessimistic solutions		0.497		1.361		0.0028	
	potential solutions		0.677		0.584		0.025	accepted
(7)	proposed constraints	0.497		0.681		0.0028		$\delta_2=0.681$
	pessimistic solutions		0.497		0.681		0.0028	
	potential solutions		0.534		0.584		0.013	accepted
(8)	proposed constraints	0.497		0.596		0.0028		$\delta_2=0.085$
	pessimistic solutions		0.497		0.596		0.0028	
	potential solutions		0.502		0.584		0.0048	accepted
(9)	proposed constraints	0.497		0.585		0.0028		$\delta_2=0.011$
	pessimistic solutions		0.497		0.585		0.0028	
	potential solutions (final solution)		0.498		0.584		0.0030	accepted
(a)	solution for (4.14) (to compare with)	0.4975		0.5844		0.00285		

* goal values belonging to optimal solutions and proposed goal constraints.
** pessimistic and potential goal values.

The relatively favourable results with the above example can be generalized without some reserves. Clearly, the number of iterations needed depends on the number of goal variables, the exact shape of the preference function and the accuracy required. Furthermore, we assumed that the decision-maker did not specify any a priori information. When this is the case, the effect on the method's convergence speed can not be determined beforehand, because such information may either speed up or slow down the process. Finally, in this example we introduced an ad hoc order of the goal variables to be changed. First, the expected growth was driven to its optimal value, then the market volatility and finally the current dividend. It may be that another order of changing the goal values may speed up the process.

To answer these questions, additional experiments are needed (and planned). In the given example, other preference functions than(4.14) must be tried. Besides the influence of the availability of a priori information should be studied. Evidently, these experiments can not be based on the above example alone. However, the final judgment on the method's merits must be passed in the practice of decision-making. To get some insight into this last question, we devote the last section of this report tosome examples of (potential) applications of I.M.G.P.

5. I.M.G.P. PUT INTO PRACTICE

In this final section we describe some (potential) applications of I.M.G.P. together with its features which may be considered to be attractive in each of these cases.

I. Production planning.

In an earlier report (Nijkamp and Spronk [1978b]) we gave an example of production planning in a brick factory. In this case, management had to choose from a convex set of feasible combinations of the quantities of two varities of bricks to be produced. For the planning period concerned management could neither define a profit function nor any other global preference function, due to uncertain market conditions and due to some technical problems in the factory. However, management stated that it wanted a 'maximum' quantity of brick variety one and a production of the second variety 'as close as possible' to a quantity of 6,000,000 units and 'certainly' over 4,000,000 units. This problem could be solved by means of I.M.G.P. It showed, that the method can be used when some of the goal variables are not to be maximized (or minimized) which may occur when some 'satisficing' target goal level exists.

II. A simple location problem.

In another recent report (Nijkamp and Spronk [1978a]) we considered an enterprise which is planning to build a new factory for the production of storage batteries. Each of the twenty candidate locations has been described in terms of their contributions to the goal variables which management considers to be relevant in this situation. Although the set of feasible actions is not convex, and although not each goal variable can be presented on a cardinal scale, I.M.G.P. can be used in this case. However, because of the non-convexity of the set of feasible actions, no advantage can be taken of the use of pessimistic solutions (see section 2). In fact, the method reduces to a systematic manner of imposing constraints on the set of feasible actions.

III. I.M.G.P. for capital budgeting.

In a subsequent report we go further into the use of I.M.G.P. for the solution of capital budgeting problems. In literature, many examples have been given of the application of 'standard' multiple goal programming in both private and public capital budgeting. Evidently, also I.M.G.P. can be used in these cases. Here, we mention some additional possibilities of I.M.G.P.

Because I.M.G.P. uses a target approach, also chance constraints can be handled in an interactive way. In this case, an additional provision is needed. This is because the method measures the deviation from a target level in a goal constraint. As such this measure is meaningless until it is translated back to the corresponding probability. This can be done by standard tables of the normal distribution or with the help of an approximation formula of the normal distribution.

Another interesting possibility of I.M.G.P. is to preselect some of the investment projects without knowledge of the discount rate to be used. Usually, the stream of cashflows generated by each project is discounted, after which the capital budgeting problem is to select, within the feasible region, those projects which together have the maximum discounted value. Clearly, the determination of the rate at which the projects are discounted, is of crucial importance. Both in theory and practice of the finance field, the discussion on the 'correct' valuation measure has not been concluded yet. A part of this problem can be circumvented by the preselection of 'inferior' projects by means of I.M.G.P. This can simply be done with the help of the potence matrix presented in section 2. As a prerequisite, the sum of the cash-flows in a particular period must be viewed as a separate goal variable. Thus, we have for each of the periods during the planning horizon a separate goal variable. Then, each of the goal variables has to be maximized, after which a pessimistic solution can be calculated. This pessimistic solution imposes minimum values on the sum of cashflows in each period by which at least some of the projects can be excluded. Some problems may arise when the projects are indivisible. These and other details are discussed in a subsequent report.

References.

1. Baum, S., and R.C. Carlson, Multi-goal Optimization in Management Science, Omega, 1974, pp. 607-623.

2. Bronsema, H.J.J., and B.V.H. van de Kieft, Enkele risicomaatstaven en beslissingscriteria voor het beleggingsprobleem onder risico: een doelprogrammeringsformulering, forthcoming in Jonkhart et al. [1978].

3. Charnes, A., and W.W. Cooper, Goal Programming and Multiple Objective Optimizations, Part I, European Journal of Operations Research, 1977, p. 39.

4. Cohon, J.L., and D.H. Marks, A Review and Evaluation of Multiobjective Programming Techniques, Water Resources Research, 1975, pp. 208-220.

5. Fishburn, P.C., Lexicographic Orders; Utilities and Decision Rules: A Survey, Management Science, 1974, p. 1442.

6. Ijiri, Y., Management Goals and Accounting for Control, North-Holland, 1965.

7. Iserman, H., The Relevance of Duality in Multiple Objective Linear Programming, in Starr and Zeleny [1977], pp. 241-262.

8. Jonkhart, M.J.L., J.W.R. Schuit and J. Spronk (eds.) Finbeldag 1978, (forthcoming), Stenfert Kroese, Leiden (Netherlands), 1978.

9. Kumar, P.C., G.C. Philippatos, and J.R. Ezzell, Goal Programming and the Selection of Portfolios by Dual-Purpose Funds, Journal of Finance, March 1978.

10. Lane, M.N., Goal Programming and Satisficing Models in Economic Analysis, Ph. D. Thesis, University of Texas, 1970.

11. Lee, S.M., Goal Programming for Decision Analysis, Auerbach, 1972.

12. Lee, S.M. and A.J. Lerro, Optimizing the Portfolio Selection for Mutual Funds, Journal of Finance, 1973, pp. 1087-1101.

13. Monarchi, D.E., J.E. Weber and L. Duckstein, An Interactive Multiple Objective Decisionmaking Aid Using Non-Linear Goal Programming, in Zeleny [1976], pp. 235-253.

14. Nijkamp, P. and J. Spronk, Goal programming for Decision-Making, Report 7709/A, Centrum voor Bedrijfseconomisch Onderzoek, Erasmus University, Rotterdam, 1977.

15. Nijkamp, P. and J. Spronk, Analysis of Production and Location Decisions by Means of Multi-Criteria Analysis, Report 7801/A, Centrum voor Bedrijfseconomisch Onderzoek, Erasmus University, Rotterdam, 1978a.

16. Nijkamp, P. and J. Spronk, Interactive Multiple Goal Programming, Report 7803/A, Centrum voor Bedrijfseconomisch Onderzoek, Erasmus University, Rotterdam, 1978b.

17. Roy, B., Problems and Methods with Multiple Objective Functions, Mathematical Programming, 1971, pp. 239-266.

18. Roy, B., From Optimisation to Multi-Criteria Decision Aid: Three Main Operational Attitudes, in Thiriez and Zionts [1976].

19. Roy, B., A Conceptual Framework for a Normative Theory of Decision-Aid, in Starr and Zeleny [1977].

20. Sharpe, W.F., A Linear Programming Algorithm for Mutual Fund Portfolio Selection, Management Science, 1967, pp. 499-510.

21. Starr, N.K. and M. Zeleny (eds.), Multiple Criteria Decision Making, TIMS Studies in Management Sciences, Vol.6., North-Holland, 1977.

22. Thiriez, H. and S. Zionts (eds.), Multiple Criteria Decision Making, Springer Verlag, Berlin, 1976.

23. Wallenius, J., Comparative Evaluation of Some Interactive Approaches to Multicriterion Optimization, Management Science, 1975, pp. 1387-1396.

24. Wallenius, J. and S. Zionts, Some Tests of an Interactive Programming Method for Multicriterion Optimization and an Attempt at Implementation, in Thiriez and Zionts [1976].

25. Zeleny, M. (ed,), Multiple Criteria Decision-Making, Springer Verlag, Berlin, 1976.

Appendix A Input Data*

Security	1959		1968
	1968		
i	E_i	B_i	D_i
I Automation & Business Services			
1	1.095	1.072	.004
2	1.391	1.500	0
3	1.311	.757	.008
4	.881	1.352	.006
II Electronics & Instrumentation			
5	1.217	1.378	0
6	1.014	1.296	.015
7	.930	1.262	.002
8	.923	1.575	.008
9	.953	1.717	.006
10	1.188	1.496	.014
11	.124	1.306	.029
12	.083	1.290	.009
III Cosmetics & Drugs			
13	.316	1.589	.013
14	.092	1.133	.019
15	.064	1.092	.017
16	.201	1.022	.023
17	.150	.851	.022
18	.154	.850	.020
19	.161	.819	.023
20	.146	.593	.014
IV Air Transport			
21	.296	1.687	.009
22	.203	1.517	.014
23	.243	1.378	.008
24	.391	1.302	.011
25	.207	1.277	.024
26	.103	1.524	.023
V Industrials			
27	.257	2.088	.032
28	.206	1.303	.014
29	.120	1.157	.004
30	.143	.985	.028
31	.093	.936	.035
VI Chemicals			
32	.199	.945	.011
33	.017	.738	.033
34	.077	.649	.032
VII Aluminium			
35	-.004	1.394	.021
36	.017	1.191	.026
37	-.006	1.011	.025
VIII Oils			
38	.120	.727	.035
39	.302	1.098	.014
40	.478	1.114	0
41	.208	.850	.022
42	.151	.752	.026
43	.077	.587	.038
IX Financial			
44	.414	1.506	0
45	.439	1.504	0
46	.119	.565	.033
X Utilities			
47	.076	1.019	.041
48	.058	.888	.028
49	.101	.610	.035
50	.070	.607	.039
51	.095	.549	.034
52	.066	.451	.048
53	.078	.403	.045

* Adopted from Lee ([1972], pp. 232-234) to whom is refered for further details on the data.

REPORTS 1978

A - General Business Economics
F - Business Finance and Portfolio Investment
M - Marketing
O - Organization, Production
ACC - Accountancy

7801/A Nijkamp, P. and J. Spronk, Analysis of production and location decisions by means of multi-criteria analysis, February 1978.

7802/M Zwan, A. van der, 'Neo-professionalisering' - integraal verslag van een onderzoek onder drie groepen beroepsbeoefenaren, maart 1978.

7803/A Nijkamp, Peter and Jaap Spronk, Interactive Multiple Goal Programming, April 1978.

7804/F Jonkhart, Marius J.L., Bond Risk Differentials, the Term Structure of Interest Rates and the Risk of Default, April 1978.

7805/A Keus, J., Investeringsbeslissingen van ondernemers in macro-economisch perspectief, april 1978, discussienota.

7806/F Herst, A.C.C., Leasen of kopen: een kwestie van druk? juni 1978.

7807/F Jonkhart, Marius J.L., Bond Risk Differentials, the term structure of interest rates and the risk of default: A correction and an extension, May 1978.

7808/M Wijst, D. van der, Enige aspekten van het onderzoek t.b.v. de psychologische positionering van produkten, mei 1978.

7809/A Moerland, P.W., Prijsgedrag in theorie en praktijk, juni 1978.

7810/F Jonkhart, Marius J.L., On the Order of Debt Claims, July 1978.

7811/F Diepenhorst, A.I., Vennootschappelijke of Particuliere Schuld-aanvraag, gevolgd door
Elkaar Compenserende Fouten, augustus 1978.

7812/F Nijkamp, Peter and Jaap Spronk, Interactive multiple goal programming: method and application, June 1978.